

# Chiral shifts in heavy-light mesons

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The mass shifts of the  $P$ -wave  $D_s$  and  $B_s$  mesons due to coupling to  $DK$  and  $BK$  channels are calculated in the coupling channel model without fitting parameters. The strong mass shifts down for  $0^+$  and  $1^{+'}$  states have been obtained, while  $1^{+''}$  and  $2^+$  states remain almost in situ. The masses of  $0^+$  and  $1^{+'}$  states of  $B_s$  mesons have been predicted.

After the experimental discovery of the  $D_s(2317)$  and  $D_s(2460)$  mesons [1], a necessity to study the chiral dynamics in heavy-light mesons became quite clear. The masses of these states proved to be much lower than expected values in ordinary quark models while their widths were surprisingly small. The problem was studied in different approaches: in relativistic quark model calculations [2]–[4], on the lattice [5], in QCD Sum Rules [6,7], in chiral models [8,9] (for reviews see also [10,11]). The masses of  $D_s(0^+)$  and  $D_s(1^{+'})$  in closed-channel approximation typically exceed by  $\sim 140$  and  $90$  MeV their experimental numbers. The main theoretical goal seems for us to understand dynamical mechanism responsible for such large mass shifts of the  $0^+$  and  $1^{+'}$  levels and explain why the position of other two levels remains practically unchanged. The importance of second fact has been underlined by S.Godfrey in [3].

The mass shifts of the  $D_s(0^+, 1^{+'})$  mesons have already been considered in a number of papers with the use of unitarized coupled-channel model [12], in nonrelativistic Cornell model [13], in semi-relativistic model with inverse heavy quark mass expansion [14], and in different chiral models [15]–[17]. Here we address again this problem with the aim to calculate also the mass shifts of the  $D_s(1^{+'})$  and  $B_s(0^+, 1^{+'})$  states and the widths of the  $2^+$  and  $1^+$  states, following the approach developed in [16], for which strong coupling to the  $S$ -wave decay channel, containing a pseudoscalar ( $P$ ) Nambu-Goldstone (NG) meson, is crucially important. Therefore in this approach principal difference exists between vector-vector ( $VV$ )

and  $VP$  (or  $PP$ ) channels. This analysis of two-channel system is performed with the use of the chiral quark-pion Lagrangian which has been derived directly from the QCD Lagrangian [18] in the frame of the Field Correlator Method (FCM) and does not contain fitting parameters, so that the shift of the  $D_s^*(0^+)$  state  $\sim 140$  MeV is only determined by the conventional decay constant  $f_K$ .

From the common point of view, due to spin-orbit and tensor interactions the  $P$ -wave multiplet of a HL meson is split into four levels with  $J^P = 0^+, 1_L^+, 1_H^+, 2^+$  [19]. Here we use the notation H(L) for the higher (lower)  $1^+$  state because a priori one cannot say which of them mostly consists of the light quark  $j = 1/2$  contribution. In fact, starting with the Dirac's  $P$ -wave levels, one has the states with  $j = 1/2$  and  $j = 3/2$ . And  $1_{L,H}^+$  eigenstates can be parameterized by introducing the mixing angle  $\phi$ :

$$\begin{aligned} |1_H^+\rangle &= \cos\phi |j = \frac{1}{2}\rangle + \sin\phi |j = \frac{3}{2}\rangle, \\ |1_L^+\rangle &= -\sin\phi |j = \frac{1}{2}\rangle + \cos\phi |j = \frac{3}{2}\rangle. \end{aligned} \quad (1)$$

In the heavy-quark (HQ) limit the states with  $j = \frac{3}{2}$  and  $j = \frac{1}{2}$  are not mixed, but for finite  $m_Q$  they can be mixed even in closed-channel approximation [8,19].

Taking the meson emission to the lowest order, one obtains the effective quark-pion Lagrangian in the form

$$\Delta L_{FCM} = - \int \psi_i^+(x) \sigma |x| \gamma_5 \frac{\varphi_a \lambda_a}{f_\pi} \psi_k(x) d^4x. \quad (2)$$

Writing the equation (2) as  $\Delta L_{FCM} = -\int V_{if} dt$ , one obtains the operator matrix element for the transition from the light quark state  $i$  (i.e. the initial state  $i$  of a HL meson) to the continuum state  $f$  with the emission of a NG meson ( $\varphi_a \lambda_a$ ). Thus we are now able to write the coupled channel equations, connecting any state of a HL meson to a decay channel which contains another HL meson plus a NG meson.

Consider a complete set of the states  $|f\rangle$  in the decay channel 2 and the set of unperturbed states  $|i\rangle$  in channel 1. One arrives at the nonlinear equation for the shifted mass:

$$m[i] = m^{(0)}[i] - \sum_f \frac{|\langle i|\hat{V}|f\rangle|^2}{E_f - m[i]}, \quad (3)$$

where  $m^{(0)}[i]$  is the initial mass, calculated in the single-channel approximation (assumed to be known),  $m[i]$  – is the final one,  $E_f$  is the energy of the final state, and the operator  $\hat{V}$  provides the transitions between the channels. Note, that in our approximation we do not take into account the final state interaction in the  $DK$  system and neglect the  $D$ -meson motion. Also, in the w.f. we neglect possible (very small) mixing between the  $D(1_{1/2}^-)$ ,  $D(1_{3/2}^-)$  states and between  $D_s(2_{3/2}^+)$ ,  $D_s(2_{5/2}^+)$  states; physical  $D_s(1^+)$  states can be mixed, though.

In subsequent analysis it is convenient to define the masses with respect to nearby threshold:  $m_{\text{thr}} = m_K + m_D$ . So, we introduce the following notations:

$$E_0 = m^{(0)}[D_s] - m_D - m_K, \quad \delta m = m[D_s] - m^{(0)}[D_s], \quad (4)$$

$$\Delta = E_0 + \delta m = m[D_s] - m_D - m_K, \quad (5)$$

where  $\Delta$  determines the deviation of the  $D_s$  meson mass from the threshold, and can be complex if a decay to  $DK$  pair is allowed. In what follows we consider unperturbed masses  $m_0(J^P)$  of the  $(Q\bar{q})$  levels as given (our results do not change if we slightly vary their position, in this way the analysis is actually model-independent).

For further calculations we should insert the explicit meson w.f. to the matrix element in (3). In our approximation for a HL meson we consider

a light  $q$  (or strange  $s$ ) quark with current (pole) mass  $m_{q,s}$  moving in the static field of a heavy antiquark  $\bar{Q}$ , and take its w.f. as a 4-spinor obeying the Dirac equation with the linear scalar potential and the vector Coulomb potential with frozen  $\alpha_s = \text{const}$ :

$$U = \sigma r, \quad V_C = -\frac{\beta}{r}, \quad \beta = \frac{4}{3}\alpha_s. \quad (6)$$

Finally, after long cumbersome calculations which are omitted here, we arrive at the following equations to determine meson masses and widths:

$$D_s(0^+) : \quad E_0[0^+] - \Delta = \tilde{\mathcal{F}}_0(\Delta),$$

$$D_s(1_L^+) : \\ E_0[1_L^+] - \Delta = \cos^2 \phi \cdot \tilde{\mathcal{F}}_0(\Delta) + \sin^2 \phi \cdot \tilde{\mathcal{F}}_2(\Delta),$$

$$D_s(1_H^+) : \\ E_0[1_H^+] - \Delta = \sin^2 \phi \cdot \tilde{\mathcal{F}}_0(\Delta) + \cos^2 \phi \cdot \tilde{\mathcal{F}}_2(\Delta),$$

$$\Gamma[1_H^+] = \sin^2 \phi \cdot \tilde{\Gamma}_0(\Delta) + \cos^2 \phi \cdot \tilde{\Gamma}_2(\Delta),$$

$$D_s(2_{3/2}^+) : \\ E_0[2_{3/2}^+] - \Delta = \frac{3}{5} \cdot \tilde{\mathcal{F}}_2(\Delta), \quad \Gamma[2_{3/2}^+] = \frac{3}{5} \cdot \tilde{\Gamma}_2(\Delta), \quad (7)$$

where  $\mathcal{F}_{0,2}$  and  $\Gamma_{0,2}$  are some universal functions; definition of those, together with calculation details, can be found in [20].

In our analysis the 4-component (Dirac) structure of the light quark w.f. is crucially important. Specifically, the emission of a NG meson is accompanied with the  $\gamma_5$  factor which permutes higher and lower components of the Dirac bispinors. For the  $j = 1/2, P$  -wave and the  $j = 1/2, S$  -wave states it is exactly the case that this “permuted overlap” of the w.f. is maximal because the lower component of the first state is similar to the higher component of the second state and vice-versa, while for the analogous overlap between  $j = 3/2, P$  -wave and the  $j = 1/2, S$  -wave states the situation is opposite. In the end, it leads to the functions  $\mathcal{F}_0$ ,  $\Gamma_0$  being much larger than  $\mathcal{F}_2$ ,  $\Gamma_2$  for almost all reasonable values of  $\Delta$ . Thus the large shift of the  $1^{+'}$  state with a concurrent small one for  $1^{+''}$  state reveals a natural explanation (see below).

Now we turn directly to the mass computations. We will take into account the following pairs of mesons in coupled channels ( $i$  refers to first (initial) channel, while  $f$  refers to second (decay) one):

$$\begin{array}{cc} i & f \\ \hline D_s(0^+) & D(0^-) + K(0^-) \\ D_s(1^+) & D^*(1^-) + K(0^-) \\ D_s(2^+) & D^*(1^-) + K(0^-) \end{array} \quad (8)$$

and analogously for  $B$ -meson case, with corresponding masses and threshold values (in MeV):

$$\begin{aligned} m_{D^+} &= 1869, & m_{D^+} + m_{K^-} &= 2363, \\ m_{D^{*+}} &= 2010, & m_{D^{*+}} + m_{K^-} &= 2504, \\ m_{B^+} &= 5279, & m_{B^+} + m_{K^-} &= 5772, \\ m_{B^*} &= 5325, & m_{B^*} + m_{K^-} &= 5819. \end{aligned} \quad (9)$$

The light quark eigenfunction is calculated numerically via Dirac equation with the following set of parameters: (the same as in our previous papers [21]):

$$\begin{aligned} \sigma &= 0.18 \text{ GeV}^2, & \alpha_s &= 0.39, \\ m_s &= 210 \text{ MeV}, & m_q &\sim 0 \text{ MeV}, \end{aligned} \quad (10)$$

The choice of  $\sigma$  and  $\alpha_s$  is a common one in the frame of the FCM approach, and the value of the light quark mass really does not influence here on any physical results because of its smallness in comparison with the natural mass scale  $\sqrt{\sigma}$ . The strange quark mass is taken from [22], where it was found from the ratio of experimentally measured decay constants  $f(D_s)/f(D)$ ; the same value can be obtained by a renormalization group evolution starting from the conventional value  $m_s(2 \text{ GeV}) = 90 \pm 15 \text{ GeV}$ .

The ultimate results of our calculations are presented in Tables 1–3. A priori one cannot say whether the  $|j = \frac{1}{2}\rangle$  and  $|j = \frac{3}{2}\rangle$  states are mixed or not. If there is no mixing at all, in this case the width  $\Gamma(D_{s1}(2536)) = 0.3 \text{ MeV}$  is obtained in [23], while the experimental limit is  $\Gamma < 2.3 \text{ MeV}$  [24] and recently in [25] the width  $\Gamma = 1.0 \pm 0.17 \text{ MeV}$  has been measured. Therefore small mixing is not excluded and here we take the mixing angle  $\phi$  slightly deviated from  $\phi = 0^\circ$  (no mixing

Table 1

$D_s(0^+)$ -meson mass shift due to the  $DK$  decay channel and  $B_s(0^+)$ -meson mass shift due to the  $BK$  decay channel (all in MeV)

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	$\delta m$
$D_s(0^+)$	2475 (30)	2330(20)	2317	-145
$B_s(0^+)$	5814(15)	5709 (15)	not seen	-105

case). Then we define those angles  $\phi$  which are compatible with experimental data for the masses and widths of both  $1^+$  states.

The large value  $\cos^2 \phi$  for the  $1_H^+(j = 1/2)$  state provides large mass shift ( $\sim 100 \text{ MeV}$ ) of this level and at the same time does not produce the mass shift of the  $1_L^+$  level, which is almost pure  $j = \frac{3}{2}$  state. We would like to stress here that the mass shifts weakly differ for  $D_s$  and  $B_s$ , or, in other words, weakly depend on the heavy quark mass.

Thus we have obtained the shifted masses  $M(B_s, 0^+) = 5710(15) \text{ MeV}$  and  $M(B_s, 1^{+'}) = 5730(15) \text{ MeV}$ , which are in agreement with the predictions in [11] and of S.Narison [7] and by  $\sim 100 \text{ MeV}$  lower than in [2],[8]. The masses of the  $2^+$  and  $1^+$  states precisely agree with experiment.

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Table 2

The  $D_s(1^+)$ ,  $D_s(2^+)$  meson mass shifts and widths due to the  $D^*K$  decay channel for the mixing angle  $4^\circ$  (all in MeV)

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	$\Gamma_{(D^*K)}^{(\text{theor})}$	$\Gamma_{(D^*K)}^{(\text{exp})}$	$\delta m$
$D_s(1_H^+)$	2568(15)	2458(15)	2460	$\times$	$\times$	-110
$D_s(1_L^+)$	2537	2535	2535(1)	1.1	$< 1.3$	-2
$D_s(2_{3/2}^+)$	2575	2573	2573(2)	0.03	not seen	-2

Table 3

The  $B_s(1^+)$ ,  $B_s(2^+)$  meson mass shifts and widths due to the  $B^*K$  decay channel for the mixing angle  $4^\circ$  (all in MeV)

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	$\Gamma_{(B^*K)}^{(\text{theor})}$	$\Gamma_{(B^*K)}^{(\text{exp})}$	$\delta m$
$B_s(1_H^+)$	5835(15)	5727(15)	not seen	$\times$	$\times$	-108
$B_s(1_L^+)$	5830	5828	5829 (1)	0.8	$< 2.3$	-2
$B_s(2_{3/2}^+)$	5840	5838	5839(1)	$< 10^{-3}$	not seen	-2

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